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Comments on AdS_2 solutions of $D = 11$ supergravity

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Abstract: We study the supersymmetric solutions of 11-dimensional supergravity with a factor of AdS_2 made of M2-branes. Such solutions can provide gravity duals of superconformal quantum mechanics, or through double Wick rotation, the generic bubbling geometry of M-theory which are 1/16-BPS. We show that, when the internal manifold is compact, it should take the form of a warped U(1)-fibration over an 8-dimensional Kähler space.

Keywords: Supergravity Models, M-Theory, AdS-CFT Correspondence.

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1. Introduction

It is one of the most intriguing issues in string theory to prove/disprove the so-called Maldacena conjecture between anti-de-Sitter gravity and conformal field theories [1]. A lot of remarkable agreements have been encountered so far, so it would be more appropriate to say one would like to determine the region of validity as precisely as possible.

This inevitably leads us to the study of *less* supersymmetric solutions of String/M-theory. The spectrum of supersymmetric solutions is rich enough to provide examples with realistic features such as confinement and asymptotic freedom (see [2] for example), and yet thanks to the power of supersymmetry we can perform explicit checks using protected quantities.

In this paper we will employ a technique which is found to be very powerful and at the same time illuminating in the search for new supersymmetric backgrounds of String/M-theory. Based on the existence of Killing spinors, one can construct various differential forms as spinor bilinears and determine the local form of the metric and the gauge fields exploiting the differential and algebraic relations between the differential forms which can be derived using the Killing spinor equations. A sample of works which make use of this technique can be found in ref. [3].

In this paper, as a sequel to the previous one [4], we analyze the consequences of supersymmetry in 11-dimensional supergravity, combined with the ansatz of AdS_2 factor, i.e. $SO(2,1)$ isometry. In particular, for simplicity, we consider pure M2-brane configurations. A convenient way of seeing them is as M2-branes wrapped on two-cycles in a Calabi-Yau 5-fold, and the readers are referred to [5] for studies along alternative avenues on similar configurations. In the present work, it will be shown that, the 9-dimensional internal manifold should take the form of a warped U(1)-fibration on a Kähler base. We also write down the 8-dimensional nonlinear partial differential equation for the curvature tensor, which is required if the supersymmetric configuration is to solve the equations of motion.

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This paper is organised as follows. In section 2, we set up the problem and derive the 9-dimensional Killing equations from the 11-dimensional one. In section 3, we consider the spinor bilinears and their derivatives to fix the local form of the metric. In section 4 we illustrate that the well-known solutions which have AdS_2 factors can be indeed cast in the form as we have presented in this paper. In section 5 we conclude with comments and discussions on further works.

2. Ansatz

In this article we consider supersymmetric solutions of D=11 supergravity, whose Lagrangian density in the bosonic sector is given as follows,

$$\mathcal{L} = R * 1 - \frac{1}{2} G \wedge *G - \frac{1}{6} C \wedge G \wedge G, \tag{2.1}$$

where C is the 3-form potential and $G = dC$.

By definition, supersymmetric backgrounds allow nontrivial solutions to the Killing spinor equation which is obtained by setting the supersymmetry transformation to zero. For D=11 supergravity, we thus have, for purely bosonic backgrounds,

$$\delta\psi_M = \nabla_M \epsilon + \frac{1}{288} \left(\Gamma_M^{\ M_1 \cdots M_4} - 8 \delta_M^{\ M_1} \Gamma^{M_2 M_3 M_4} \right) G_{M_1 \cdots M_4} \epsilon = 0, \tag{2.2}$$

where ϵ is the supersymmetry parameter which is a Majorana spinor in $D = 11$ and $\Gamma_M,\ M = 0, 1, \cdots, 10$ are the gamma matrices satisfying $\{\Gamma_M, \Gamma_N\} = 2 g_{MN}$.

Now we restrict ourselves to a specific class of solutions which have an AdS_2 factor. We can write the metric as follows,

$$ds^2 = e^{2A} ds^2(AdS_2) + g_{ab} dx^a dx^b, \quad a, b = 1, 2, \cdots, 9\,. \tag{2.3}$$

A is the warp factor and a function of x^a only, and it does not depend on the coordinates of AdS_2. Technically we are performing a dimensional reduction on AdS_2 to get a 9 dimensional system from the 11 dimensional supergravity, but the Euclidean space defined by x^a will be referred to as *internal*. Without loss of generality we set the radius of AdS_2 to 1, i.e. the scalar curvature is 2.

Furthermore, we consider *electric* ansatz for the field strength G, and take

$$G = \mathrm{Vol}_{AdS_2} \wedge F. \tag{2.4}$$

Note that most generally one can also consider turning on the 4-form fields in the internal 9 dimensional space. Our setup amounts to the consideration of pure M2-brane configurations, and among other things trivializes the effect of the cubic interaction of the gauge field C. The equation of motion and the Bianchi identity for C are reduced to the following expressions for the two-form field F in the 9 dimensional internal space,

$$d(e^{-2A} * F) = 0, \quad dF = 0. \tag{2.5}$$

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We now need to introduce the basis for the gamma matrices which is convenient for the dimensional decomposition we have chosen to consider. With tangent space indices, they are

$$\Gamma_\mu = \gamma_\mu \otimes 1, \quad \mu = 0, 1, \tag{2.6}$$

$$\Gamma_a = \gamma_3 \otimes \gamma_a, \quad a = 2, \cdots, 10, \tag{2.7}$$

where γ_μ's are the 2 dimensional gamma matrices, while γ_a's denote 9 dimensional ones. For simplicity we take the basis where all γ_μ and γ_a are real, and of course Γ_M's are also real matrices.

As the ansatz for the Killing spinor ϵ, we assume its 2 dimensional part satisfies

$$\bar{\nabla}_\mu \varepsilon = \frac{a}{2}\gamma_\mu \varepsilon, \quad a = \pm 1. \tag{2.8}$$

Here $\bar{\nabla}$ denotes the covariant derivative on a unit-radius AdS_2. There is an alternative to this equation, i.e. $\bar{\nabla}_\mu \varepsilon = \frac{ai}{2}\gamma_\mu\gamma_3\varepsilon$, but in 2 dimensions this just corresponds to the parity inversion so for definiteness we choose eq. (2.8). We can then take the 11 dimensional spinor $\epsilon = \varepsilon \otimes \eta + c.c.$, where η is a 9 dimensional spinor.

Combining the ingredients given above, it is straightforward to derive the 9 dimensional Killing spinor equations, which can be presented as follows.

$$aie^{-A}\eta + \partial\!\!\!/ A\eta - \frac{1}{6}e^{-2A} F\!\!\!/\eta = 0, \tag{2.9}$$

$$\nabla_a\eta + \frac{1}{24}e^{-2A}\left(\gamma_a{}^{bc}F_{bc} - 4F_{ab}\gamma^b\right)\eta = 0. \tag{2.10}$$

In the following we will analyze how the above equations restrict the local form of the 9-dimensional internal metric.

3. Spinor bilinears and the consequences of supersymmetry

We now consider the various spinor bilinears made out of η, and exploit the Killing spinor equations to find the local form of the metric. We can consider the real-valued differential forms such as

$$f = \eta^\dagger\eta, \tag{3.1}$$

$$K = \eta^\dagger\gamma_a\eta\, dx^a, \tag{3.2}$$

$$Y = \frac{ai}{2}\eta^\dagger\gamma_{ab}\eta\; dx^a \wedge dx^b. \tag{3.3}$$

For instance, with the zero-form f one can easily verify that, using eq. (2.9) and eq. (2.10),

$$\nabla_a(\eta^\dagger\eta) = \frac{1}{3}e^{-2A}F_{ac}\eta^\dagger\gamma^c\eta \tag{3.4}$$

$$= \partial_a A\eta^\dagger\eta, \tag{3.5}$$

implying that one can set $\eta^\dagger\eta = e^A$. We proceed in the same way and find that $\nabla_a K_b + \nabla_b K_a = 0$, i.e. K defines a Killing vector in the internal space, and

$$d(e^A K) = F + Y. \tag{3.6}$$

For the two-form Y, we get simply

$$dY = 0. \tag{3.7}$$

Now we can choose the coordinate system where $K = \partial_\psi$ and the metric of the internal space is

$$ds^2 = e^{2\phi}(d\psi + B)^2 + g_{ij}dy^i dy^j \quad i, j = 1, 2, \cdots, 8. \tag{3.8}$$

ϕ, B are respectively a scalar and a vector field defined on the 8 dimensional space $\mathcal{M}_8$ with coordinates y^i.

Now we claim that, when the 9 dimensional internal space is compact, η is a chiral spinor on $\mathcal{M}$, and as a result $\phi = A$. We will need to make use of the following identities which hold for an arbitrary Dirac spinor η in 9-dimensions.

$$(\eta^T\eta)^2 = (\eta^T\gamma_a\eta)^2, \tag{3.9}$$

$$|\eta^T\eta| - |\eta^T\gamma_a\eta|^2 = 2(\eta^\dagger\gamma_a\eta)^2 - 2(\eta^\dagger\eta)^2, \tag{3.10}$$

whose derivation can be found in ref. [6].

We consider the spinor bilinear $\eta^T\eta$, which is complex-valued. From the Killing equations we get

$$\partial_a(e^{-A}\eta^T\eta) = aie^{-2A}\,\eta^T\gamma_a\eta. \tag{3.11}$$

For $D = e^{-A}\eta^T\eta$, we thus have $(\nabla_a D)^2 = -e^{-4A}(\eta^T\gamma_a\eta)^2 = -e^{-2A}D^2$. One can also show that, from the Killing equations eq. (2.9) and eq. (2.10),

$$\begin{aligned}\nabla^2(e^{-A}\eta^T\eta) &= ai\nabla^a(e^{-2A}\eta^T\gamma_a\eta)\\ &= -2e^{-3A}\eta^T\eta,\end{aligned} \tag{3.12}$$

i.e. $\nabla^2 D + 2e^{-2A}D = 0$. From these relations, it is easy to see that $\nabla^2(D^{-1}) = 0$ provided D is not zero. If the internal space is compact, this is possible only if D is constant, but it means $D = 0$, so we have a contradiction. We thus conclude $\eta^T\eta = \eta^T\gamma_a\eta = 0$, and from eq. (3.10) η is chiral on $\mathcal{M}_8$ and we have $K^2 = e^{2\phi} = e^{2A}$.

Obviously the above argument requires the internal space should be compact, and A should not show a singular behavior. But in the next section we will show that, for the important class of solutions such as $AdS_4 \times SE_7$ and the bubbling geometry of ref. [7], even though the internal manifolds are not compact, the solutions can be rewritten in the manner we will conclude in this section. We guess that it might be possible to improve our proof for the chirality of η, without assuming compact internal space.

Now that η is chiral, Y is a closed two-form in $\mathcal{M}_8$, which can be used to define an almost complex structure. In order to see whether this complex structure is integrable or not, one needs to check the exterior derivative of the complex-valued $(4,0)$-form Ω, defined as

$$\Omega_{abcd} = \eta^T\gamma_{abcd}\eta. \tag{3.13}$$

The chirality of η again restricts Ω to be a four-form on $\mathcal{M}_8$, and as usual with $SU(n)$-structures, J, Ω satisfy

$$\mathrm{Vol}_{\mathcal{M}_8} = \frac{e^{-4A}}{24}J\wedge J\wedge J\wedge J = \frac{e^{-2A}}{16}\Omega\wedge\bar{\Omega}, \quad J\wedge\Omega = 0. \tag{3.14}$$

Using the Killing equations one obtains

$$d(e^A\Omega) = -aie^{-A}K \wedge \Omega. \tag{3.15}$$

For $\omega \equiv e^A e^{-ia\psi}\Omega$ and using $K = e^{2A}(d\psi + B)$, we have

$$d\omega = -aiB \wedge \omega, \tag{3.16}$$

which is an 8 dimensional equation on $\mathcal{M}_8$. From the general result of $SU(n)$-structures and the classification of torsion classes, we arrive at the conclusion that the complex structure given by Y is integrable and $dB = \mathcal{R}$ is the Ricci form of the Kähler manifold $\mathcal{M}_8$. Considering $Y^2 \sim (\eta^\dagger\eta)^2 \sim e^{2A}$ and $\omega^2 \sim e^{2A}|\eta^T\eta|^2 \sim e^{4A}$ when evaluated using the metric g, it is the rescaled metric $\bar{g}_{ij} = e^A g_{ij}$ which is Kähler.

One can rephrase eq. (3.6) to get

$$F = \bar{F} + 3e^A dA \wedge K, \tag{3.17}$$

$$e^{3A}\mathcal{R} = \bar{F} + Y, \tag{3.18}$$

where $\bar{F}$ is the two-form field F restricted to $\mathcal{M}_8$. Now if we contract eq. (2.9) with $\eta^\dagger$, we have $\bar{F}_{ij}Y^{ij} = -6e^{-2A}$. We thus have the expression for the scalar curvature R of $\mathcal{M}_8$ whose Kähler metric is given as $\bar{g}$,

$$R = 2e^{-3A}. \tag{3.19}$$

Consideration of other spinor bilinears does not generate independent equations, but we need to impose the Bianchi identity and the equation of motion for F, to guarantee that the supersymmetric configuration really satisfies all the equations of motion [10]. It turns out that $dF = 0$ is a consequence of supersymmetry, as can be easily seen from eq. (3.6) and eq. (3.7). Using the equation of motion for F, $\nabla^a(e^{-2A}F_{ab}) = 0$, and eq. (3.4), one can derive the following equation for the scalar curvature of $\mathcal{M}_8$.

$$\Box R - \frac{1}{2}R^2 + R_{ij}R^{ij} = 0. \tag{3.20}$$

We conclude that, our ansatz of supersymmetric AdS_2 requires the metric should locally be written as

$$ds^2 = e^{2A}ds^2_{AdS_2} + e^{2A}(d\psi + B)^2 + e^{-A}\bar{g}_{ij}dy^i dy^j, \tag{3.21}$$

where $\bar{g}$ defines a Kähler metric, and dB is the Ricci form of the 8 dimensional base space with coordinates y^i.

4. Examples of AdS_2 solutions and their Kähler geometry

In this section we will illustrate that several classes of well-known solutions which include an AdS_2 factor can be indeed written in the form given as eq. (3.21).

The simplest case is obtained when we take $\mathcal{M}_8 = S^2 \times T^6$. Being related to the curvature scalar of $\mathcal{M}_8$, A is a constant, and the eleven dimensional spacetime becomes $AdS_2 \times S^3 \times T^6$, where the radius of S^3 is twice as large as that of AdS_2. This solution is a well-known example, which is given as the near-horizon limit of three M2-branes intersecting over a point. As such, the preserved supersymmetry is in fact 1/4 for this solution.

We next consider $AdS_4 \times SE_7$ solutions where SE_7 is a 7 dimensional Sasaki-Einstein manifold. They can be considered as the near-horizon limit of M2-brane solutions when put on a singularity of Calabi-Yau 4-folds. As such, in general these solutions are 1/8-BPS.

It is well known that in canonical form, any Sasaki-Einstein manifolds can be written as a U(1)-fibration over a Kähler-Einstein manifold. For SE_7, using the standard convention,

$$ds^2 = (d\alpha + \frac{\sigma}{4})^2 + ds^2_{KE_6}, \tag{4.1}$$

where $d\sigma = \mathcal{R} = 8J$. R is the Ricci form and J is the Kähler form of the 6 dimensional Kähler-Einstein manifold. In the following we will rewrite the 11-dimensional metric in a form where the 8-dimensional Kähler structure is manifest.

$$\begin{aligned} ds^2 &= \frac{1}{4} ds^2_{AdS_4} + ds^2_{SE_7} \\ &= \frac{1}{4}\left[\cosh^2\rho\, ds^2_{AdS_2} + d\rho^2 + \sinh^2\rho d\phi^2\right] + \left(d\alpha + \frac{\sigma}{4}\right)^2 + ds^2_{KE_6} \\ &= \frac{1}{4}\cosh^2\rho\, ds^2_{AdS_2} + \frac{1}{4}\cosh^2\rho\left(d\phi + 2\frac{d\tilde{\alpha}+\sigma/4}{\cosh^2\rho}\right)^2 \\ &\quad + \frac{2}{\cosh\rho}\left(\frac{\cosh\rho}{2}(\frac{d\rho^2}{4} + ds^2_{KE_6}) + \frac{\sinh^2\rho}{2\cosh\rho}\left(d\tilde{\alpha} + \frac{\sigma}{4}\right)^2\right), \end{aligned} \tag{4.2}$$

where we put $\alpha \to \tilde{\alpha} + \phi/2$.

Compared to the general form of the solution given in eq. (3.21), evidently we expect that the 8-dimensional metric

$$ds^2 = \frac{\cosh\rho}{2}\left(\frac{d\rho^2}{4} + ds^2_{KE_6}\right) + \frac{\sinh^2\rho}{2\cosh\rho}\left(d\tilde{\alpha} + \frac{\sigma}{4}\right)^2 \tag{4.3}$$

should be Kähler, and the Ricci-form is given as

$$\mathcal{R} = d\left(2\frac{d\tilde{\alpha}+\sigma/4}{\cosh^2\rho}\right), \tag{4.4}$$

with the scalar curvature

$$R = \frac{16}{\cosh^3\rho}, \tag{4.5}$$

and finally, eq. (3.20) should be satisfied.

In order to check this, it is most efficient to construct the almost Kähler form J and the $(4,0)$-form Ω, and compute their exterior derivatives. A reasonable guess for J is

$$J = \frac{1}{4}\sinh\rho d\rho \wedge \left(d\tilde{\alpha} + \frac{\sigma}{4}\right) + \frac{\cosh\rho}{2} J_{KE_6}. \tag{4.6}$$

One can readily check that $dJ = 0$. From the complex structure given by J, the (4,0)-form Ω is given as

$$\Omega = \frac{\cosh\rho}{8}\left(\cosh\rho d\rho + 2i\sinh\rho\left(d\tilde{\alpha} + \frac{\sigma}{3}\right)\right) \wedge \Omega_{KE_6}. \tag{4.7}$$

One can also check that, as required by eq. (4.4),

$$d\Omega = \frac{2i}{\cosh^2\rho}(d\tilde{\alpha} + \frac{\sigma}{4}) \wedge \Omega. \tag{4.8}$$

And eq. (3.20) is indeed satisfied.

The next example is the 1/2-BPS bubbling geometry of M-theory giant gravitons obtained in [7]. This class of solutions describe generic 1/2-BPS operators of M2-brane or M5-brane conformal field theories. From superalgebra arguments, such configurations should possess $SO(3) \times SO(6)$ symmetry, which are realized as a $S^2 \times S^5$ factor in the metric. S^2 can be treated as a Wick-rotated AdS_2 to fit into our result. Another comment in order is that now the canonical Killing vector made out of the Killing spinor becomes time-like, so our solutions can describe magnetic, or M5-branes, as well as electric or M2-brane configurations. The solutions are summarized as follows.

$$\begin{aligned}
ds_{11}^2 &= -4e^{2\lambda}(1+y^2e^{-6\lambda})(dt+V_idx^i)^2 + \frac{e^{-4\lambda}}{1+y^2e^{-6\lambda}}[dy^2 + e^D(dx_1^2+dx_2^2)] \\
&\quad +4e^{2\lambda}d\Omega_5^2 + y^2e^{-4\lambda}d\tilde{\Omega}_2^2 \\
G &= F \wedge d\tilde{\Omega}_2^2 \\
e^{-6\lambda} &= \frac{\partial_y D}{y(1-y\partial_y D)} \\
V_i &= \frac{1}{2}\epsilon_{ij}\partial_j D \quad \text{or} \quad dV = \frac{1}{2} *_3 [d(\partial_y D) + (\partial_y D)^2 dy] \\
F &= dB_t \wedge (dt+V) + B_t dV + d\hat{B} \\
B_t &= -4y^3e^{-6\lambda} \\
d\hat{B} &= 2 *_3 [(y\partial_y^2 D + y(\partial_y D)^2 - \partial_y D)dy + y\partial_i\partial_y D dx^i] \\
&= 2\tilde{*}_3[y^2(\partial_y \frac{1}{y}\partial_y e^D)dy + ydx^i\partial_i\partial_y D]
\end{aligned} \tag{4.9}$$

where $i, j = 1, 2$ and $*_3$ is the three dimensional ϵ symbol of the metric $dy^2 + e^D dx_i^2$ and $\tilde{*}_3$ is the flat space ϵ symbol. The function D satisfies the equation

$$(\partial_1^2 + \partial_2^2)D + \partial_y^2 e^D = 0. \tag{4.10}$$

In order to see the Kähler structure the metric of the equation let us first rewrite (4.9) as follows,

$$\begin{aligned}
ds_{11}^2 &= y^2e^{-4\lambda}d\tilde{\Omega}_2^2 + 4e^{2\lambda}[(d\alpha + \frac{\sigma}{3})^2 + ds_{KE_4}^2] \\
&\quad -4e^{2\lambda}(1+y^2e^{-6\lambda})(dt+V)^2 \\
&\quad +\frac{e^{-4\lambda}}{1+y^2e^{-6\lambda}}[dy^2 + e^D(dx_1^2+dx_2^2)],
\end{aligned} \tag{4.11}$$

where the 4-dimensional Kähler-Einstein manifold is CP^2, which provides the Kähler-Einstein base for the trivial example of 5-dimensional Sasaki-Einstein manifold S^5. In fact, as far as supersymmetry is concerned, any 4-dimensional Kähler-Einstein manifold would suffice as long as it satisfies $\mathcal{R} = 6J = d\sigma$.

If we define $\alpha = \tilde{\alpha} + t$, then the metric can be rewritten as

$$
\begin{aligned}
ds_{11}^2 = & \; y^2 e^{-4\lambda} d\tilde{\Omega}_2^2 - 4y^2 e^{-4\lambda}[dt - \frac{e^{6\lambda}}{y^2}(d\tilde{\alpha} + \frac{\sigma}{3} - (1 + y^2 e^{-6\lambda})V)]^2 \\
& + \frac{4}{ye^{-2\lambda}} \left\{ \frac{e^{6\lambda}}{y}(1 + y^2 e^{-6\lambda})(d\tilde{\alpha} + \frac{\sigma}{3} - V)^2 \right. \\
& \left. + \frac{1}{4}\frac{ye^{-6\lambda}}{1 + y^2 e^{-6\lambda}}[dy^2 + e^D(dx_1^2 + dx_2^2)] + yds_{KE_4}^2 \right\}.
\end{aligned}
\tag{4.12}
$$

Now we are ready to identify the Kähler structure of the M-theory bubbling solution. The Kähler form of the 8-dimensional base space is given by

$$
J = -\frac{1}{4}\partial_y(e^D)dx_1 \wedge dx_2 + \frac{1}{2}dy \wedge (d\tilde{\alpha} + \frac{\sigma}{3} - V) + yJ_{KE_4}, \tag{4.13}
$$

where J_{KE_4} is the Kähler form of KE_4. One can easily show that J is closed, and for the $(4, 0)$-form given as

$$
\Omega = 4ye^{D/2}\left(\partial_y D\, dy + 2i(d\tilde{\alpha} + \frac{\sigma}{3} - V)\right) \wedge (dx_2 + idx_1) \wedge y\Omega_{KE_4}, \tag{4.14}
$$

and also we indeed have, as expected from the twisting of the U(1)-fibration in the metric,

$$
d\Omega = 2i\left(\frac{e^{6\lambda}}{y^2}(d\tilde{\alpha} + \frac{\sigma}{3}) - \frac{e^{6\lambda} + y^2}{y^2}V\right) \wedge \Omega. \tag{4.15}
$$

5. Discussions

In this work we have studied supersymmetric M2-brane configurations which have a factor of AdS_2. They can be interpreted as the near-horizon limit of M2-branes, whose worldvolume is wrapped on a 2-cycle in a Calabi-Yau 5-fold. In general, they are thus 1/16-BPS. It turns out that the internal 9-dimensional manifold should take a form of U(1)-fibration whose base manifold is given by a Kähler manifold. There is a restriction on the Kähler base imposed by supersymmetry: we have found a Laplace-like equation for the scalar curvature and Ricci tensor, as given in eq. (3.20).

The result presented in this article is amusingly very similar to the result of [4], where pure D3-brane configurations with AdS_3 are studied. In that case, the internal manifold is 7-dimensional, which again takes the form of warped U(1)-fibration on a 6-dimensional Kähler manifold. The Kähler base cannot be arbitrary, it has to satisfy a nonlinear partial differential equation for the curvature.

It is certainly of great interest to find new AdS solutions, by directly trying to solve eq. (3.20). In fact, recently a new class of AdS_3 solutions in IIB supergravity has been presented [8]. The authors first studied AdS_3 solutions of 11-dimensional supergravity, and then, through T-duality operations, obtained AdS_3 solutions of IIB supergravity, where only the five-form fluxes are turned on. This is of course very similar to the discovery of the celebrated Sasaki-Einstein solutions $Y^{p,q}$ as part of AdS_5 solutions in IIB supergravity [9]. In particular, it was argued that the new solutions can be indeed written in the form as

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presented in [4]. The relevant 6-dimensional Kähler manifold takes a form of S^2-bundle over a 4-dimensional Kähler-Einstein manifold. It will be very interesting to introduce such a concrete ansatz, and solve eq. (3.20). We expect, as is the case with $Y^{p,q}$, the metric of the Kähler base might be in general *not* complete, but the entire 9-dimensional internal manifold can be made complete. We plan to present the new solutions and the global analysis in a future publication.

Another interesting direction is to interpret our solutions as generalized bubbling geometry. For the case of $AdS_5 \times S^5$ solutions in IIB supergravity, the 1/2-BPS bubbling solutions given in [7] can be described in terms of a distribution function in the phase space of 1-dimensional free fermions. For M-theory, determining the solutions is instead reduced to solving a Toda equation in 3-dimensions, and we believe the dynamics of 1/2-BPS operators must be encoded therein. The supergravity solutions dual to less supersymmetric giant graviton operators are considered for 1/8-BPS in ref. [4] and for 1/4-BPS in ref. [11]. They both conclude that the 10 dimensional solution is based on a Kähler space which is 6 and 4-dimensional, respectively. Naturally one expects they originate from the symplectic structure of the eigenvalue dynamics. Likewise, eq. (3.20) can be interpreted as the equation governing the dynamics of generic supersymmetric operators of M-branes conformal field theory. One important feature we have been ignoring in this paper is the global property and the boundary conditions of the solutions. Since our analysis is general enough to encompass the 1/2-BPS fluctuations of less supersymmetric conformal field theories on M-branes, as well as less supersymmetric fluctuations of maximal conformal field theories on M-branes, we will first need to fix the boundary condition according to the conformal field theory we are interested in, and then solve eq. (3.20) to find gravity duals to BPS operators.

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